MATH 603

Book Review:
“Bourbaki, A Secret Society of Mathematicians”

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Author: Maurice Mashaal
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**Overview**

#### *Bourbaki: A Secret Society of Mathematicians* is written by Maurice Mashaal, a French journalist and currently editor-in-chief of *Pour la Science* in Paris. In this book Mashaal gives a very detailed history, background, sequence of events, as well as a very informative and gripping illustrations and photographs of many mathematicians, their works, and events that influenced their lives. The book begins with the story of how the group was formed. In 1935 an enthusiastic group of nine French mathematicians from L’Ecole Normale Superieure have decided that it was time to write an analysis textbook that was adequate for the teaching of the subject, as they all agreed that then current book was of poor quality. It was Edourard Goursat’s *Cours d’analyse* that created all this unhappiness and a drive to get a better text. To help with the process, they have established some quirky rules, some of which were: the group would hold three conferences a year (some spanned over 2 weeks), no outside person could know who the active members were, and all of its members were to retire at the age of 50. The author then goes on to describe the meetings, giving a good number of anecdotal evidence of conversations, ideas that were thrown around. The basic strategy of the group was to divide the chapters of their desired textbook and give each member a chapter to work on. Then, once the first draft was complete, it took quite a long time to settle on the approval of the chapter as every member of the group had to agree with everything that was written. This has created conflicts and tension, and some of the original members had to leave because they could not agree with some of the popular views of the majority of the group. Overall, Bourbaki earned fame and influence in part because of the scientific strength of its members. All of the members of Bourbaki were very good or even excellent mathematicians with great productivity outside of the Bourbaki projects (p. 19). At its culmination, and a total of 7000 pages of written mathematics was produced by the group. However, it was not an original piece of work, rather than a collection of what the members of the group perceived as a modern view of mathematics, relevant and up-to-date. The name of the book, or in fact we would call it series now is “Elements de Mathematique” first came out in 1939-1940. Since then it had a number of editions and was printed in many copies, with translations in several languages. Next, the reader is invited to learn more about the name and some of the reasons why it was picked, with references to general Charles Bourbaki, active and well-known French general of the nineteenth century, and the prank played on freshmen students of L’Ecole Normale Superieure (also referred to as Normaliens) when Andre Weil (one of the founders of Bourbaki) gave a pseudo-lecture concluding with “Bourbaki’s Theorem”. This lecture became legendary, and added to the legend was the assertion that one of the students claimed to understand it completely (p. 23).

**Implications in Philosophy and Nature of Mathematics**

In Chapter 4 the author gives the reader a little taste for the mathematics that was written in the Bourbaki’s Elements. The topics they engaged in were: set theory, algebra, general topology, functions of one variable, topological vector spaces, integration, commutative algebra, groups, differential and analytical varieties, and spectral theory. There is very limited rigorous mathematics present on these pages, but rather little snippets and popular diagrams. However, the general ideas about each of the fields above are well-communicated, summarized nicely and accessible for audience with some knowledge of university mathematics. This gives a very good set up and playing field for what comes next. Chapter 5 opens with the following statement: “An intellectual construction endowed with profound unity, a hierarchy of abstract structure built on a foundation of axioms: this is how Bourbaki viewed mathematics. The group’s beliefs gained many followers” (p.70). Not only the group wrote voluminous and ambitious mathematical treatise, but also they propagated a certain vision of mathematics. Their philosophy revolved around three main notions: the unity of mathematics, the axiomatic method, and the study of structures. Mashaal argues that the members of Bourbaki were one of the first to entertain the idea that geometry, algebra, analysis, and number theory are no longer separate topics, and modern mathematics research applies to all domains. This is very well summarized with the following question: “In short, is modern mathematics a single mathematique or many mathematiques?” (p. 72). This creates some food for thought, some of which is brought up earlier in the book: the study of mathematics has become a vast field, with hundreds of thousands of papers published each year, thus meaning that there simply could not be a single person who is an expert in mathematics as a whole, yet things are interconnected and united under a common mathematique. This can only really mean one thing that mathematics is a completely social entity with a necessary component of collaboration and dependence on other experts to help with one’s research. Next, the book goes on to talk about the axiomatic method of mathematics as viewed by the Bourbaki members. This modern method does not try to define basic concepts that the theory in question is trying to discuss. These basic concepts are treated as abstract entities whose nature and concrete meaning are insignificant. Only the relations between the fundamental entities defined by the axioms are important. The properties deduced from such a formal theory are completely general, since they could apply to a very different set of objects, as long as axioms for that set of objects are the same (p.75). The author goes on to show an example of this with the definition of a group, as really anything at all that satisfies the three properties (four for an abelian group) can be classified as a group. This is very different from the Euclidean axiomatic principles where all the objects are clearly defined and there is a geometric, thus visual representation of each of them. This is where concept image and concept definition are starting to depart a little, as now there is no unique visual representation of the concept, as it is more general and many things, fundamentally different come together under the same classification. Mashaal gives a very good example of how the construction of these axioms happens: the mathematician starts by studying a certain set of objects, and then develops a set of axioms based on these objects. He has in mind a certain well-defined mathematical objects, and then after a careful examination and checks of the proof, there happen to be a very small number of properties of the objects in question that played any sort of role on the proof. Thus these properties become axioms, oftentimes being able to reduce further, as more refinement is happening to the newly discovered set of objects (p. 76). Another fruitful idea is explained on page 79: “With the axiomatic method and these structures (groups, rings, fields, order, neighbourhoods, limits, continuity), Bourbaki paints a picture of mathematical universe where the organizing principle is a hierarchy of structures progressing from simple to complex, and from general to specific”.

**Implications in Mathematics Education**

“New Math in the Classroom” is the title of Chapter 10 of the book. It begins with a snippet of a grade eight textbook where a definition of a graduated line is given. Looking at it for several minutes one can start to recognize the notion of slope and a slope-intercept form of the equation of a line. Let’s put it this way, if I was to give this to my grade 10 class (where the equation of a line lives in our curriculum), I would probably be working for one more month. According to the author, Bourbaki had played a role in this more in France than anywhere else, but it was a sufficient involvement to get into the issue deeper. Triggered by the space advancements of the Soviet Union, the Western Block has turned to re-evaluation of the science and mathematics education and curricula. The idea of introducing the axiomatic method from very early stages of primary and secondary education was the number one idea that the reformers of the time were entertaining. Due to the work of Jean Piaget, in which he saw an analogy between the mental structures created when child learns mathematics and the mother-structures discussed by Nicolas Bourbaki, the New Math was supported and implemented. The basic idea was to emphasize rigour in definitions, theorems, and proof-writing while deemphasizing all numerical, algebraic, and trigonometric calculations. The reform unfortunately went too far and met very large opposition of complaints from the public and professional levels. Quotes such as: “New Math’s concepts are defined without the reference to reality, and that it is impossible to reason with these concepts and to find interest in them, learning them is a test of memory that poisons intelligence” have contributed to the perceived failure of the reform and overall public disapproval. There is yet another great quote on page 143: “The goal of mathematics is not to prove rigorously things that everyone knows, but it is to find rich results and then, in order to make sure they are true, to prove them”. At the end of this chapter, there appears to be a curious paragraph that has little to do with Bourbaki, but perhaps is the opinion of the author. He introduces an image of our current mathematics curriculum as Sierpinski triangle. The idea is that things keep disappearing from the curriculum, even though they do not pose a substantial difficulty to the students, for example vector spaces, number theory and geometry. Further suggestion is made that the abandonment of New Math may have perhaps involved throwing the baby out with the bathwater. “Students are not really taught to reason, and mathematics inspires as much fear, lack of interest, as even hatred as ever.

**Conclusion**

This is a very well written book, given that it is largely about mathematics and mathematicians. I would rate it as accessible by general public, with some university knowledge of mathematics and history of mathematics. The author is very good at painting the environment and people he is describing. It is also very well illustrated with pictures and diagrams of both people and their work. Excellent historical snippets and stories are offered, which are entertaining to read. This makes them very hard to forget. Not only the book describes the lives and work of the members of Bourbaki, but also goes into speculations of the reasons and possible ideas they might have been thinking about at that time. Furthermore, it is particularly interesting to read about the changes and influence the group had on the nature and philosophy of mathematics, as well as on mathematics education. The book is fairly easy to read if one just wants to get a historical perspective for a leisure pastime, or careful examination of philosophical issues. A closer look at the latter will provide very fruitful to anybody who is interested in the matter. I have enjoyed the read very much, as it gave me a good inside into the life and minds of mathematicians of the previous century, as most of our professional interactions are mostly with characters and their work from more past times. This was a very nice change from that.