**Reasoning Applied to Geometry of Polygon Angles** Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date:

Vocabulary: Polygon: closed plane figure (meaning many-sided flat shape with connected sides)

Regular polygon: all the sides and angles are the same size

Irregular polygon: a polygon that is not regular

Diagonal: line from a vertex to another vertex that is not a side

Interior angle: inside angle between two sides

Exterior angle: angle formed by extending one side

Central angle: angle formed by connecting two radii to the centre

Polygon types:

Triangle: 3-sided Quadrilateral: 4-sided

Pentagon: 5-sided Hexagon: 6-sided

Heptagon or Septagon: 7-sided Octagon: 8-sided

Nonagon: 9-sided Decagon: 10-sided

Regular Polygons: These can be created by placing equally spaced points (vertices) on a circle:

Central Angles in Polygons: Connect each vertex (polygon corner) to the centre of the circle:

[The triangle and pentagon are done for you. Now complete the other three shapes.]

Use logic now with the fact that there are 360° in a full circle at the centre to complete the table:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name of Regular Polygon | Number of Sides | Measure of Central Angle | Measure of base $L $of isosceles  | Measure of interior $L$ of polygon | Sum of the measures of the interior angles |
| Triangle | 3 | 360÷3=120 | (180-120)÷2=30° | 2x30°=60° | 3x60°=180° |
| Square | 4 |  | (180-90°)÷2=45° | 2x45°=90° | 4x90°=360° |
| Pentagon |  |  |  |  |  |
| Octagon |  |  |  |  |  |
| Nonagon |  |  |  |  |  |
| Dodecagon | 12 |  |  |  |  |
| n-sided polygon |  |  |  |  |  |

 Check your last cell’s result with teacher:

Irregular Polygons: Require drawing diagonals from one vertex to the others:

Use the logical fact of triangle angle sums equaling 180°. Finish the last two

|  |  |  |  |
| --- | --- | --- | --- |
| **Name of Polygon** | **Number of Sides** | **Number of non-overlapping triangles** | **Sum of the measures of the interior angles of shape** |
| Triangle | 3 | 1 | 180° x 1 = 180° |
| Quadrilateral | 4 | 2 | 180° x 2 = 360° |
| Pentagon |  |  |  |
| Septagon |  |  |  |
| Nonagon |  |  |  |
| 100-sided polygon |  |  |  |
| n-sided polygon |  |  |  |

 Check your last cell’s result with teacher:

From your last two tables’ conclusions:

 **The sum of the interior angles of an n-sided polygon is**

Since there are n equal interior angles in a **regular polygon**, describe how you would find the measure of each interior angle: (Include a formula in your description)

**Check Up on Skills and Understanding**: Solve the following showing work

1. What is the sum of the interior angles in an 11-sided polygon?
2. How many sides does a polygon have whose interior angle sum is 3960?
3. How many sides does a regular polygon have whose interior angle is 170°?
4. What is the interior angle of a 20-sided regular polygon?
5. Determine the values of n, p and y in the diagrams below:

2y+30°

y+20°

3y-30°

y-10°

 a) b) c)

120°

120°

140°

140°

n

170°

100°

p

BIG IDEAS: If the total interior angle sum of a regular polygon is found by 180(n-2) then the size of each congruent (equal) interior angle is $\frac{180\left(n-2\right)}{n}$ . We can also find any exterior angle to the regular polygon by subtracting an interior angle from the straight angle 180°.

$$Exterior angle=180- \frac{180\left(n-2\right)}{n}= \frac{180n}{n}- \frac{180\left(n-2\right)}{n}= \frac{180n-180n+360}{n}= \frac{360}{n}$$

Therefore: The sum of the measures of the exterior angles of an n-sided regular polygon **is 360°**

**Now solve the intentional practice and concept development questions of the next pages**:

**Concept Development and Intentional Practice of Polygon angles**: Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Complete the table below of the following regular polygons:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name of Regular Polygon | Number of Sides | Measure of Central Angle | Measure of base $∠$ of isosceles  | Measure of interior $L$ of polygon | Sum of the measures of the interior angles |
| Hexagon |  |  |  |  |  |
| Heptagon |  |  |  |  |  |
| Decagon |  |  |  |  |  |
| Tridecagon | 13 |  |  |  |  |
|  |  |  |  | 108° |  |
| x-sided polygon |  |  |  |  |  |
|  |  |  |  |  | 3240° |

1. Complete the table to determine the measures of the interior angle sum

|  |  |  |
| --- | --- | --- |
| **Number of sides** | **Number of non-overlapping triangles** | **Interior angle sum** |
| 9 |  |  |
| 13 |  |  |
| 21 |  |  |
| 42 |  |  |
| 6n |  |  |
| h+2 |  |  |

1. Determine the number of sides of each polygon given the interior angle sum:

a) 720° b) 2340° c) 2880°

1. Determine the number of sides of a regular polygon given the interior angle:

a) 156° b) 160° c) 165°

1. Determine the number of sides of a regular polygon given the exterior angle:

a) 40° b) 5° c) 14.4°

Further Concept Development: Date: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. The interior angle of a regular polygon is 4 times the exterior angle. What is the size of each of the central angles? (Include a diagram to help organize your thinking)
2. How many diagonals does a 10-sided polygon have? Remember: “If a problem is too big, don’t do it! Do smaller problems until you perceive a pattern(s) to help solve the bigger problem.”

8.) What is the sum of X + a + b? 9.) Given that all smaller central angles are

X+40

X-10

2X-40

X

X+20

X+10

a

b

 equal, calculate the size of angle ABH.

A

B

H

C

D

E

G

F

O

P

10.) A 360-sided polygon can be used to simulate a circle on a computer screen. What is the size of the interior angle? Show several methods of calculating it: